

# Anomaly Mediation and Fixed Point in Partially $N = 2$ Supersymmetric Standard Models

Wen Yin<sup>1</sup>

*Department of Physics, Tohoku University, Sendai 980-8578, Japan*

## Abstract

To explain the tension between the observed Higgs boson mass and the experimental deviations from the Standard Model (SM) prediction in flavor physics, especially the experimental anomaly of the muon anomalous dipole moment (muon  $g - 2$ ), we study partially  $N = 2$  supersymmetric (SUSY) extensions of the SM (partially  $N = 2$  SSMs). In this kind of model, an  $N = 2$  SUSY sector is sequestered from the SUSY breaking due to  $SO(2)_R$  symmetry at the tree-level. We show that the low energy physics in the  $N = 2$  sector is controlled by a fixed point and hence approximately UV insensitive. Moreover at this fixed point, the tachyonic slepton problem of anomaly mediation is always solved. In a concrete partially  $N = 2$  SSM, the muon  $g - 2$  anomaly is explained within the  $1\sigma$  level error with  $\mathcal{O}(100)$  TeV cosmologically favored gravitino. We also propose some new dark matter candidates as a natural consequence of partially  $N = 2$  SSMs.

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<sup>1</sup>email: yinwen@tuhep.phys.tohoku.ac.jp

# 1 Introduction

The supersymmetric (SUSY) extension of the Standard Model (SM) is a leading candidate of the physics beyond the electroweak (EW) scale, which has more consistency than the SM does with gauge unified theory (GUT), string theory etc. In the minimal supersymmetric SM (MSSM), the recently measured Higgs boson mass,  $\sim 125$  GeV [1], implies a heavy stop. This is because the Higgs boson mass is predicted to be below  $\sim 90$  GeV at the tree-level and should be raised via quantum corrections. With negligible stop-mixing, the stop masses of  $\mathcal{O}(10)$  TeV are required to generate a large enough radiative correction[2]. On the other hand, in flavor physics, some discrepancies of physical quantities between their experimental values and the SM predictions are observed [3]. This suggests that some of the sparticles, including sfermions, are light enough to have proper SUSY contributions in explaining them. For example, the measured muon anomalous magnetic moment (muon  $g-2$ ) at the Brookhaven E821 experiment [4] deviates from the SM prediction [5, 6] more than  $3\sigma$  level experimental and theoretical errors. In the MSSM, to explain this discrepancy at the  $1\sigma$  level, a smuon, with mass around 300 GeV, is typically required [7].

If true, these facts imply that the sfermion spectrum is splitting. SUSY SM models with splitting spectra, especially with heavy stops and light smuons, is well-studied [8].

According to [9], a splitting sfermion spectrum can be generated in a partially  $N = 2$  SUSY model. The authors show a SUSY breaking in  $N = 2$  SUSY, which preserves a part of  $SU(2)_R$  symmetry, e.g. an  $SO(2)_R$  symmetry, does not generate soft scalar masses for hypermultiplets. In a partially  $N = 2$  SUSY model, other than this  $N = 2$  SUSY sector ( $N = 2$  sector) we have an  $N = 1$  SUSY sector ( $N = 1$  sector). Then such a SUSY breaking lead to suppressed hypermultiplet scalar soft masses contrary to those of ordinary chiral multiplets in the  $N = 1$  sector. Therefore, if the lighter sfermions, say smuons, are within the  $N = 2$  sector, while the stops are within the  $N = 1$  sector, we can obtain the splitting sfermion spectrum.

In this paper, we consider an effective theory with an  $SO(2)_R$  symmetry imposed in the  $N = 2$  sector at the tree-level. In particular, we suppose that the  $N = 2$  SUSY partners in the  $N = 2$  sector do not decouple until the SUSY scale,  $\mathcal{O}(10)$ TeV, namely we consider the partially  $N = 2$  supersymmetric extensions of the MSSM

(partially  $N = 2$  SSMs). Further imposing the decoupling of the gauge multiplets from the SUSY breaking, the  $SO(2)_R$  symmetry requires the sequestering of the  $N = 2$  sector from the SUSY breaking at the tree-level, namely the tree-level SUSY breaking gaugino and scalar soft masses in the  $N = 2$  sector are vanishing. Also the  $N = 2$  SUSY relation [10] between the Yukawa couplings and the gauge couplings in the  $N = 2$  sector is required at the tree-level.

The SUSY breaking soft masses in the  $N = 2$  sector are generated via radiative corrections which are dominantly anomaly mediation effect [11, 12]. At the quantum level, the Yukawa couplings in the  $N = 2$  sector are controlled by an infra-red (IR) fixed-point. We find that the tachyonic slepton problem [11] in the original anomaly mediation scenario is always solved at this fixed-point due to the non-trivially large Yukawa couplings. In realistic cases, due to the large gauge couplings induced by introducing the additional particles, these Yukawa couplings are approximated to be the fixed point values. Hence at the low energy, the anomaly mediation induced (anomaly induced) masses are positive and approximately ultra-violet (UV) insensitive. This fact implies the precision of the imposed  $SO(2)_R$  symmetry at the tree-level is not so essential in our prediction of the low energy  $N = 2$  Yukawa couplings and the anomaly induced masses. On the other hand, in the  $N = 1$  sector the scalar soft masses, including the stop masses, are allowed to be  $\mathcal{O}(10)\text{TeV}$  at the tree-level, and hence the Higgs boson mass can be explained.

Using a concrete partially  $N = 2$  SSM, we show indeed that the tachyonic slepton problem is solved at the SUSY scale,  $\mathcal{O}(10)\text{TeV}$ . In the  $N = 2$  sector at this scale, an Yukawa coupling is approximated by its fixed point value. Also the mass of an MSSM particle in the  $N = 2$  sector is approximated by only one free parameter, the gravitino mass. Moreover, we show the muon  $g - 2$  anomaly can be explained by the light smuons with the gravitino mass around  $100\text{TeV}$ . In this model we have several cosmologically favored points, and we propose new natural candidates of the dark matter (DM). Some ordinary phenomenological problems in the MSSM are alleviated.

This paper is organized as follows. In Sec.2.1, we give an introduction to partially  $N = 2$  SUSY models by using a toy model, and we will illustrate how the  $SO(2)_R$  symmetry sequester the  $N = 2$  sector from the SUSY breaking at the tree-level. Then we will show an IR fixed point where the tachyonic slepton problem is automatically solved. In Sec.3, we relate the toy model with partially  $N = 2$  SSMs, and

show analytically that in a partially  $N = 2$  SSM, the IR behavior are controlled by this fixed point. We also give an concrete example of partially SSMs with its several phenomenological and cosmological aspects. In Sec.4, we conclude this paper, and in Sec.5 we discuss about some unsolved problems and their possible solutions.

## 2 Partially $N = 2$ Supersymmetric Model

### 2.1 Introduction to a Partially $N = 2$ Supersymmetric Model

We would like to consider an  $U(N_c)$  Yang-Mills theory with partially  $N = 2$  SUSY defined by the following Lagrangian as a toy model.

$$\mathcal{L} = \int d^4\theta (K_{N=2} + K_{N=1}) + \int d^2\theta W_{N=2} + \int d^2\theta W_{mass} + \int d^2\theta W_H \quad (1)$$

$$+ \frac{1}{2} \left( \int d^2\theta \text{tr} [\mathcal{W}^c \mathcal{W}^c] + \frac{1}{2} \int d^2\theta \mathcal{W} \mathcal{W} \right) + h.c.$$

$$K_{N=2} = 2 \text{tr} [G^\dagger e^{-2g_c V_c} G e^{2g_c V_c}] + |\phi|^2 + \sum_i^{N_F} \left( L_i^\dagger e^{2g_c V_c + 2Y_i g V} L^i + \bar{L}_i e^{-2g_c V_c - 2Y_i g V} \bar{L}^{i\dagger} \right), \quad (2)$$

$$W_{N=2} = - \sum_i^{N_F} \left( \sqrt{2} \omega_i g_c \bar{L}_i G L_i + \sqrt{2} \tilde{Y}_i g \bar{L}_i \phi L_i \right). \quad (3)$$

$$K_{N=1} = \sum_i^{N_F} S_i^\dagger e^{2g_c V_c + 2Y_i g V} S^i + \sum_i^{N_f} H_i^\dagger e^{2g_c V_c + 2Y_i^H g V} H^i \quad (4)$$

$$W_{mass} = -M_G \text{tr} [G^2] - M \phi^2 - \sum_i^{N_F} M_i \bar{L}_i S^i \quad (5)$$

where  $\mathcal{W}^c$  ( $\mathcal{W}$ ) and  $V_c$  ( $V$ ) are the field strength and the corresponding gauge multiplet of the  $SU(N_c)$  ( $U(1)$ ) with gauge coupling,  $g_c$  ( $g$ ), respectively. Here,  $W_H = W_H(H_i, S_j)$  is an arbitrary superpotential of  $H_i$  and  $S_j$ .  $L_i, \bar{L}_j, G$ , and  $\phi$  have the Yukawa couplings dominantly in Eq.(3).

#### 2.1.1 $N = 2$ SUSY limit and Two Sectors

With the decoupling of the multiplets,  $H_i$  and  $S_j$ ,

$$\omega_i = 1, \tilde{Y}_i = Y_i, \quad (6)$$

Sector	$N = 2$ sector				$N = 1$ sector	
Chiral multiplets	$G$	$\phi$	$L_i$	$\bar{L}_i$	$S_i$	$H_i$
$N = 2$ partner	$V$	$V_c$	$\bar{L}_i$	$L_i$	/	/
$N = 2$ multiplet	vector	vector	hyper-	hyper-	/	/
$SU(N_c)$ representation	adjoint	1	$r_i$	$\bar{r}_i$	$r_i$	$r_i^H$
$U(1)$ charge	0	0	$Y_i$	$-Y_i$	$Y_i$	$Y_i^H$
$i_{\max}$	/	/	$N_F$	$N_F$	$N_F$	$N_f$

Table. 1: Particle contents of a partially  $N = 2$  SUSY model Eq.(1). The representation  $r_i$  is allowed to be unity. “Vector” and “hyper-” denote the corresponding particles are within the vector and hyper- multiplets, respectively.

is the  $N = 2$  SUSY limit [10]. To see the property at this limit, let us focus on some of the Yukawa couplings from  $W_{N=2}$  and  $K_{N=2}$ :

$$\mathcal{L} \supset \sum_{i=1}^{N_F} \left( -i\sqrt{2}\tilde{L}_i^\dagger (g_c\lambda_c + Y_i g\lambda) \cdot \psi_{Li} - \sqrt{2}\tilde{L}_i (\omega_i g_c \psi_G + \tilde{Y}_i g \psi_\phi) \cdot \psi_{Li} \right). \quad (7)$$

Here,  $\psi_G$  ( $\psi_\phi$ ) and  $\lambda_c$  ( $\lambda$ ) are the gaugini, the fermionic component of  $G$  ( $\phi$ ), and gaugino in the adjoint representation of the  $SU(N_c)$  ( $U(1)$ ) gauge groups. We can find a symmetry with Eq.(6), called  $SU(2)_R$  symmetry, under which  $\{i\lambda_c, \psi_G\}$ ,  $\{i\lambda, \psi_\phi\}$  and  $\{\tilde{L}_i, \tilde{L}_i^\dagger\}$  are doublets while  $\psi_{Li}$  is a singlet. Since this symmetry relates two  $N = 1$  multiplets, in  $N = 2$  SUSY we have two kinds of enlarged multiplets, e.g.  $\{V, \phi\}$  makes an  $N = 2$  vector multiplet, and  $\{L_i, \bar{L}_i\}$  makes an  $N = 2$  hypermultiplet (hypermultiplet).

Since the first and second terms of Eq.(7) come from the Kähler and super- multiplets, respectively, this symmetry mixes the terms in these two potentials. Moreover, it mixes the kinetic terms of the  $N = 1$  multiplets in the  $N = 2$  vector multiplet, namely the Kähler potential and the SUSY gauge kinetic term are mixed.

We define that the particles existing in this  $N = 2$  SUSY limit compose an  $N = 2$  sector, namely,  $V_c, V, G, \phi$ , and sets of  $L_i, \bar{L}_i$  are the components. On the other hand the multiplets decoupling at this limit are defined to compose the  $N = 1$  sector, namely sets of  $S_i$  and  $H_j$  are the components.

We will use the definitions here to describe the phenomenon in the  $N = 2$  sector, even with explicit breaking of  $N = 2$  SUSY.

The particle contents and their profiles are summarized in Table.1.

### 2.1.2 Radiative Corrections

In this toy model of Eq.(1), the 1-loop RG equations for the dimensionless couplings and the anomalous dimensions for the chiral multiplets are given as follows at the 1-loop level [13].

$$\frac{d}{dt}g_c \equiv \beta_c = \frac{1}{16\pi^2}g_c^3(F_2 + F_1 - 2N_c), \quad \frac{d}{dt}g \equiv \beta = \frac{1}{16\pi^2}g^3(f_2 + f_1), \quad (8)$$

$$\frac{d}{dt}(g_c w_i) = g_c \omega_i (\gamma_G + \gamma_{Li} + \gamma_{\bar{Li}}), \quad (9)$$

$$\frac{d}{dt}(g \tilde{Y}_i) = g \tilde{Y}_i (\gamma_\phi + \gamma_{Li} + \gamma_{\bar{Li}}), \quad (10)$$

where

$$\gamma_G = \frac{1}{16\pi^2} \left( \sum_i^{N_F} 2T(r_i) \omega_i^2 - 2N_c \right) g_c^2, \quad \gamma_\phi = \frac{1}{16\pi^2} \sum_i^{N_F} 2d(r_i) \tilde{Y}_i^2 g^2, \quad (11)$$

$$\gamma_{Li} = \gamma_{\bar{Li}} = \frac{1}{16\pi^2} \left( 2(\omega_i^2 - 1)C(r_i)g_c^2 + 2(\tilde{Y}_i^2 - Y_i^2)g^2 \right), \quad (12)$$

$$F_2 \equiv \sum_i^{N_F} 2T(r_i), \quad F_1 \equiv \sum_i^{N_F} T(r_i) + \sum_i^{N_f} T(r_i^H), \quad (13)$$

$$f_2 \equiv \sum_i^{N_F} 2d(r_i)Y_i^2, \quad f_1 \equiv \sum_i^{N_F} d(r_i)Y_i^2 + \sum_i^{N_f} d(r_i^H)(Y_i^H)^2. \quad (14)$$

Here  $t = \log(\frac{\mu_{RG}}{\text{GeV}})$  is the logarithm of the renormalization scale,  $\mu_{RG}$ .  $T(r)$ ,  $C(r)$  and  $d(r)$  denote the Dynkin index, the quadratic Casimir invariant and the dimension of the representation  $r$ , respectively. Hence,  $F_2$  and  $f_2$  are the sums of Dynkin indices of  $\text{SU}(N_c)$  and  $\text{U}(1)$  in the  $N = 2$  sector, respectively.

At the  $N = 2$  limit where  $f_1 = 0, F_1 = 0$  with Eq.(6), we can find  $\frac{d}{dt}\omega_i = \frac{d}{dt}\tilde{Y}_j = \gamma_{Li, \bar{Li}} = 0$ , and hence Eq.(6) and the vanishing of Eq.(12) are satisfied at any scale.

## 2.2 $N = 2$ SUSY Breaking generates Splitting Mass Spectra

Suppose that we add an  $N = 2$  SUSY breaking sector<sup>1</sup> with  $N = 2$  SUSY and a messenger sector with superpotential,

$$W_{SB} = \sqrt{2}\phi_m (\tilde{Y}_m g \phi + \omega_m g_c G + Z) \phi_m, \quad (15)$$

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<sup>1</sup>For example, we can consider an abelian  $N = 2$  vector multiplet,  $\{Z, V_Z\}$  with  $N = 2$  Fayet-Iliopoulos term,  $W = \xi Z$ . Then,  $Z$  obtain an  $F$ -term spontaneously. If the messenger has the charge of this abelian gauge group, then Eq.(15) is obtained.

to Eq.(1). Here  $\{\phi_m, \bar{\phi}_m\}$  is an  $U(N_c)$  charged hypermultiplet as the messenger, and  $Z = M + \theta^2 F_Z$  is a SUSY breaking field. At the  $N = 2$  SUSY limit,

$$\tilde{Y}_m = Y_m, \quad \omega_m = 1, \quad (16)$$

where  $Y_m$  is the  $U(1)$  charge of  $\phi_m$ . Integrating out this messenger, we obtain the soft masses expressed as [21]

$$m_i^2 = \frac{1}{2} \left| \frac{F_Z}{Z} \right|^2 \left( \frac{d}{dt} \gamma_i^- - \frac{d}{dt} \gamma_i^+ \right). \quad (17)$$

where  $i$  denotes  $L_i, H_j$  etc. and  $+(-)$  denotes the value evaluated before (after) the integration of the messenger. Now, suppose that the relations, Eq.(6) and Eq.(16), are satisfied even in the presence of the  $N = 1$  sector at the messenger scale  $M^2$ , then from Eqs.(8)-(12), we can find,

$$m_{L_i}^2 = m_{\bar{L}_i}^2 = 0 \quad (18)$$

at the 2-loop level, while  $m_{H_i}^2$  and  $m_{S_i}^2$  are generated as the usual gauge mediation scenario. Hence a splitting soft mass spectrum is generated at the scale  $M$ .

In fact, these vanishing soft scalar masses are the consequence of the symmetry,  $SU(2)_R$ . From the non-vanishing expectation value,  $F_Z$ , in Eq.(15), the potential acquires,

$$\delta V = F_Z \tilde{\phi}_m \tilde{\bar{\phi}}_m + h.c. = (\tilde{\phi}_m^*, \tilde{\bar{\phi}}_m)^* \cdot (\Re[F_Z] \sigma_1 - \Im[F_Z] \sigma_2) \cdot (\tilde{\phi}_m^*, \tilde{\bar{\phi}}_m)^T, \quad (19)$$

where  $\sigma_1$  and  $\sigma_2$  are the Pauli matrices while  $T$  denotes the transpose. Therefore, non-vanishing  $F_Z$  breaks SUSY, but preserves an  $SO(2)_R$  symmetry. Obviously this  $SO(2)_R$  also mixes the Kähler and the super- potentials and mixes the Kähler potential and the SUSY gauge kinetic terms.

From this  $SO(2)_R$  symmetry, we can find that the wave function renormalization of a hypermultiplet should be a real constant independent of  $Z$  and  $Z^\dagger$ . If not, we will have  $Z$  and  $Z^\dagger$  in the coefficient in front of the first term of Eq.(7) and hence the  $SO(2)_R$  requires the same coefficient for the second term. This would lead to the inconsistency that the superpotential contains a non-holomorphic term. Therefore, the soft scalar mass of the hypermultiplet is forbidden by the  $SO(2)_R$  symmetry[9].

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<sup>2</sup>This condition might be realized at the spontaneously partially breaking scale of the  $N = 2$  SUSY [14, 9] or at the compactification scale an extra-dimensional theory with branes [15]. In [16], we will provide some of the theoretical backgrounds based on the latter possibility.

From this symmetry argument, we can conclude that if the  $\text{SO}(2)_R$  in the  $N = 2$  sector remains as a good symmetry just after the SUSY breaking, suppressed soft scalar masses of the hypermultiplets can be generated.

### 2.3 Soft Masses in the Partially $N = 2$ SUSY Model

The mechanism reviewed in the previous subsection does not depend on the detail of a SUSY breaking mechanism, but is a consequence of a symmetry it preserves. Hence, from now on we do not specify a SUSY breaking mechanism, but will suppose that such an  $\text{SO}(2)_R$  preserving SUSY breaking takes place at a high energy scale, say  $M_{GUT} \sim 2 \times 10^{16} \text{GeV}$ . After integrating out such an  $N = 2$  SUSY breaking sector, we can obtain an effective theory with the  $\text{SO}(2)_R$  symmetry in the  $N = 2$  sector at the tree-level at  $M_{GUT}$ . Namely, at  $M_{GUT} \sim 2 \times 10^{16} \text{GeV}$ , the following condition should be satisfied in Eq.(1) at the tree-level:

$$\omega_i(M_{GUT}) = 1, \quad \tilde{Y}_i(M_{GUT}) = Y_i, \quad (20)$$

and

$$m_{L_i, \bar{L}_i}^2(M_{GUT}) = 0. \quad (21)$$

In addition, we will suppose that the gauge sector is decoupled from the SUSY breaking, which might be a consequence of the fact that the SUSY breaking fields are all charged under some symmetries. Hence, the gaugino masses, namely the term as  $\frac{Z}{M_{GUT}} \text{tr}[\mathcal{W}_c \mathcal{W}_c]$ , are negligible or forbidden. Furthermore, in our scenario we can find that the soft scalar masses of  $G$  and  $\phi$  are also vanishing at the tree-level. This is because the  $\text{SO}(2)_R$  symmetry also mixes the kinetic terms of a gaugino and a gaugino, so that the only terms that can generate these soft scalar masses, as  $\delta K = \frac{Z}{M_{GUT}} \text{tr}[G^\dagger G] + h.c.$  via kinetic normalization, are, in turn, negligible or forbidden. Also, a vanishing  $S - (D-)$ term of  $\text{U}(1)$  is supposed. Then, at the tree-level,

$$m_G^2(M_{GUT}) = m_\phi^2(M_{GUT}) = 0, \quad (22)$$

$$A - \text{terms}|_{M_{GUT}} = 0, \quad M_c(M_{GUT}) = M(M_{GUT}) = 0, \quad (23)$$

and

$$S \equiv \sum_i^{N_f} d(r_i^H) m_{H_i}^2 Y_i^H + \sum_i^{N_F} d(r_i) (m_{L_i}^2 - m_{\bar{L}_i}^2 + m_{S_i}^2) Y_i = 0, \quad (24)$$



are satisfied, where  $m_G^2$  and  $m_\phi^2$  are the scalar soft masses of  $G$  and  $\phi$ , respectively, while,  $M_c$  and  $M$  are the gaugino masses of  $SU(N_c)$  and  $U(1)$ , respectively. Namely, the  $N = 2$  sector are sequestered from the SUSY breaking at the tree-level.

Since the  $N = 1$  sector explicitly breaks the  $SO(2)_R$ , the radiative corrections alters Eqs.(20) and (23) at the 1-loop level, while Eqs.(21) and (22) at the 2-loop level. The radiative corrections are not only from the RG running but also from anomaly mediation [11, 12].

The 2-loop RG equation of a scalar mass has the dominant term characterized by,

$$\beta_{m_{Li,\bar{L}i,G}^2}^{(2)} \supset (\frac{1}{16\pi^2})^2 m_{H_i,S_i}^2, \quad (25)$$

while the anomaly induced masses are defined as

$$m_{Li,\bar{L}i,G,\phi}^2 = \frac{1}{2} m_{3/2}^2 \frac{d}{dt} \gamma_{Li,\bar{L}i,G,\phi}, \quad (26)$$

where  $m_{3/2}$  is the gravitino mass. Since  $m_{H_i,S_i}^2$  should be generated via the explicit breaking of the  $SO(2)_R$ , we may expect<sup>3</sup>

$$m_{H_i,S_i}^2 \ll m_{3/2}^2. \quad (27)$$

Hence we will study the anomaly mediation effect in our scenario which can be a dominant contribution to the  $N = 2$  sector scalar soft masses and gaugino masses, given by

$$M_c = m_{3/2} \frac{\beta_c}{g_c}, \quad M = m_{3/2} \frac{\beta}{g}. \quad (28)$$

In general, it is well-known that a scalar with anomaly induced mass might be tachyonic, which is called the tachyonic slepton problem. This is the case when the scalar has a dominant gauge interaction of which the beta-function is positive, and hence Eqs.(26) is negative. In the MSSM, the sleptons with negligible Yukawa couplings suffer from this problem. However, we will show that in our set-up this problem is naturally solved.

## 2.4 A Fixed Point of the $N = 2$ Yukawa Couplings

One of the fascinating point in the anomaly mediation scenario is the UV insensitivity of Eqs.(26) and (28). Namely, the anomaly induced masses in the  $N = 2$  sector at the SUSY scale,  $m_{SUSY} \sim \mathcal{O}(10)\text{TeV}$ , are also given by the formula, Eq.(26), with gauge

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<sup>3</sup>We may also use the ordinary sequestering to satisfy this condition [11]

and Yukawa couplings evaluated at  $m_{SUSY}$ . From this reason, in this subsection, we will study the RG behavior of these couplings analytically at the 1-loop level.

We have shown in Sec.2.1.2 that, in the limit of  $N = 2$  SUSY, Eqs.(6) is satisfied at any scale. This fact implies Eq.(6) is a fixed point of the parameter space characterized by  $\{\omega_i, \tilde{Y}_i\}$ . We will show that in the presence of the degree of  $H_i$  and  $S_i$ , an IR fixed point characterized by  $\{\bar{\omega}_i, \bar{\tilde{Y}}_i\}$  also exists, but moves to a different point in the parameter space.

Firstly, we divide  $N_F$  into  $N_F^s$  and  $N_F - N_F^s$ , where  $N_F^s$  is the total number of  $SU(N_c)$  singlets of  $L_i$  for convenience. Without loss of generality, we can rearrange the indices of the  $N = 2$  sector, such that the superfields labelled by  $i = 1 \sim N_F^s$  are singlets, while those of  $i = N_F^s + 1 \sim N_F$  are not. We also divide  $f_2$  into

$$f_2^s \equiv \sum_i^{N_F^s} 2Y_i^2, \quad f_2^{ns} \equiv \sum_{i=N_F^s+1}^{N_F} 2d(r_i)Y_i^2. \quad (29)$$

In the calculation we will assume,

$$C(r_i)g_c^2 \sim C(r_i)g_c^2\omega_i^2 \gg Y_i^2 g^2 \sim \tilde{Y}_i^2 g^2. \quad (30)$$

This condition is analogue to the realistic case where  $(4/3)g_3^2$  or  $(3/4)g_2^2 \gg Y_i^2 g_Y^2$  is satisfied for a squark or left-handed slepton. Here,  $g_Y, g_2$ , and  $g_3$  are the gauge coupling constants of  $U(1)_Y, SU(2)_L$ , and  $SU(3)_c$ , respectively.

A solution of the vanishing condition for  $\frac{d}{dt}\omega_i, \frac{d}{dt}(g\tilde{Y}_i)$  is,

$$\bar{\omega}_i^2 - 1 = \frac{d(r_i)}{2T(r_i)} \frac{F_1}{\sum_{i=N_F^s+1}^{N_F} d(r_i) + 2(N_c^2 - 1)} + \mathcal{O}\left(\frac{g^2 Y_i^2}{g_c^2 C(r_i)}\right) \quad (i > N_F^s), \quad (31)$$

$$g^2 \bar{\tilde{Y}}_i^2 = 0 \quad (i > N_F^s), \quad (32)$$

The second equation is obtained by neglecting terms of  $\mathcal{O}(g^2)$  while considering  $(f_1 + f_2)g^2$ . This approximation is also based on the realistic case where  $(f_1 + f_2)g_Y^2 > g_2^2, g_3^2$  due to the large coefficient  $f_1 + f_2 \geq 11$ .

Hence we find a general relation,

$$\bar{\omega}_i^2 - 1 > 0. \quad (33)$$

In particular,  $\bar{\omega}_i^2$  can largely deviate from 1 when  $F_1$  is sizable, namely when the charged particles in the  $N = 1$  sector dominate over those in the  $N = 2$  sector.

Now we would like to check whether this fixed point is an IR fixed point.

Rewriting Eqs.(9) and (10) in terms of  $\delta\omega_i^2 \equiv \omega_i^2 - \bar{\omega}_i^2$  and  $\delta\tilde{Y}_i^2 \equiv \tilde{Y}_i^2 - \bar{Y}_i^2$ , we obtain

$$\frac{d}{dt}\delta\omega_i^2 \sim \sum_{j=N_F^s+1}^{N_F} \delta\omega_j^2 \frac{g_c^2}{8\pi^2} A_{ij}, \quad \frac{d}{dt}(g^2\delta\tilde{Y}_i^2) \sim \sum_{j=N_F^s+1}^{N_F} g^2\delta\tilde{Y}_j^2 \frac{g_c^2}{8\pi^2} B_{ij} \quad (i > N_F^s) \quad (34)$$

at the leading order, where  $A_{ij}$  and  $B_{ij}$  are the positive-definite matrices:

$$A_{ij} = 2\bar{\omega}_i^2 (T(r_j) + 2C(r_j)\delta_{ij}), \quad B_{ij} = 4(\bar{\omega}_i^2 - 1)C(r_i)\delta_{ij}. \quad (35)$$

Using the analytic solutions of Eqs.(8), we can solve Eq.(34) and obtain,

$$\delta\omega_i^2(\mu_{RG}) \sim \sum_{j=N_F^s+1}^{N_F} \left[ \left( \frac{1}{\frac{\alpha_{\text{GUT}}}{4\pi}} \right)^{\frac{A}{F_1+F_2-2N_c}} \right]_{ij} \delta\omega_j^2(M_{GUT}), \quad (36)$$

$$\delta(g^2\tilde{Y}_i^2)|_{\mu_{RG}} \sim \sum_{j=N_F^s+1}^{N_F} \left[ \left( \frac{1}{\frac{\alpha_{\text{GUT}}}{4\pi}} \right)^{\frac{B}{F_1+F_2-2N_c}} \right]_{ij} \delta(g^2\tilde{Y}_j^2)|_{M_{GUT}} \quad (i > N_F^s). \quad (37)$$

Since the exponents of Eq.(36) are positive-definite if  $g_c$  is asymptotic non-free, i.e.  $F_1 + F_2 - 2N_c > 0$ , Eqs.(31) and (32) represent an IR fixed-point. This IR fixed point is easily reached, when  $\alpha_{\text{GUT}} \equiv \frac{g_c^2}{4\pi}$  and  $\bar{\omega}_i$  in Eq.(31) is large enough.

Applying the same procedure for  $\text{SU}(N_c)$  singlets which are labelled by  $i \leq N_F^s$ , we can get the IR fixed point behavior for the remaining  $N = 2$  Yukawa couplings,  $\tilde{Y}_i$ . Since the fixed point value of a set of  $g^2 Y_i^2$  ( $i > N_F^s$ ) is zero, we approximate non-singlets of  $\text{SU}(N_c)$  of  $L_i, \bar{L}_i$  as the fields in the  $N = 1$  sector in this calculation. We obtain,

$$\bar{Y}_i^2 - Y_i^2 = \frac{1}{2} \left( \frac{f_1 + f_2^{ns}}{N_F^s + 2} \right) \quad (i \leq N_F^s). \quad (38)$$

Also shown is a general relation,

$$\bar{Y}_i^2 - Y_i^2 > 0. \quad (39)$$

From the convergence expressed by

$$\frac{d}{dt}\delta\tilde{Y}_i^2 \sim \sum_j^{N_F^s} \delta\tilde{Y}_j^2 \frac{g^2}{8\pi^2} C_{ij} \quad (i \leq N_F^s) \quad (40)$$

with

$$C_{ij} = \bar{Y}_i^2 (1 + \delta_{ij}), \quad (41)$$

we obtain

$$\delta\tilde{Y}_i^2(\mu) \sim \sum_j^{N_F^s} \left[ \left( \frac{1}{\frac{\alpha_{\text{GUT}}}{4\pi}} \right)^{\frac{C}{f_1+f_2}} \right]_{ij} \delta\tilde{Y}_j^2(M_{GUT}) \quad (i \leq N_F^s). \quad (42)$$

This implies that a set of  $\tilde{Y}_i$  always represents an IR fixed-point due to the positive exponent. Supposing that  $f_1 + f_2 \sim \mathcal{O}(10)$  as in the realistic case, we find that the attractive force is rather weaker than that of Eq.(36), due to the suppressed exponent.

## 2.5 A Solution to the Tachyonic Slepton Problem

Let us assume that at  $m_{\text{SUSY}}$ , the  $N = 2$  Yukawa couplings,  $\{\tilde{w}_i, \tilde{Y}_i\}$ , in the parameter space is approximated by the fixed point value  $\{\bar{w}_i, \bar{Y}_i\}$  in Eqs.(31), (32) and (38). In the next section, we will show that this assumption is satisfied in partially  $N = 2$  SSMS. Substituting Eqs.(8) and (12) with the  $N = 2$  Yukawa couplings at the fixed point into Eq.(26), the anomaly induced mass for the partially  $N = 2$  SUSY model is given by

$$m_{L_i, \bar{L}_i}^2|_{\text{fp}} \sim \frac{1}{16\pi^2} m_{3/2}^2 (\bar{w}_i^2 - 1) C(r_i) \frac{d}{dt} g_c^2 + \frac{1}{16\pi^2} m_{3/2}^2 (\bar{Y}_i^2 - Y_i^2) \frac{d}{dt} g^2 \quad (43)$$

$$= \frac{1}{16\pi^2} m_{3/2}^2 \left( \frac{(N_c^2 - 1) F_1}{\sum_{i=N_F^s+1}^{N_F} d(r_i) + 2(N_c^2 - 1)} g_c \beta_c - 2Y_i^2 g \beta \right) \quad (44)$$

$$\sim \frac{1}{16\pi^2} m_{3/2}^2 \left( \frac{(N_c^2 - 1) F_1}{\sum_{i=N_F^s+1}^{N_F} d(r_i) + 2(N_c^2 - 1)} g_c \beta_c \right), \quad (45)$$

for non-singlets of  $\text{SU}(N_c)$ , while

$$m_{L_i, \bar{L}_i}^2|_{\text{fp}} = \frac{1}{16\pi^2} m_{3/2}^2 (\bar{Y}_i^2 - Y_i^2) \frac{d}{dt} g^2 = \frac{1}{16\pi^2} m_{3/2}^2 \left( \frac{f_1 + f_2^{ns}}{N_F^s + 2} \right) g \beta, \quad (46)$$

for singlets of  $\text{SU}(N_c)$ .

In the case of positive  $\beta$  and  $\beta_c$ , which would lead to negative scalar mass squares in the original anomaly mediation scenario, we can find that the Eq.(45) and Eq.(46) are always positive in our scenario.

On the other hand, the scalar components of  $H_i$  and  $S_i$  might suffer from negative anomaly induced mass squares, while this is not a problem as they can have tree-level mass squares which are positive. Also, the negative anomaly induced mass squared

for  $G$  is not so problematic since it is allowed to have a tree-level SUSY mass as in Eq.(5).

In addition, this scenario seems to inherit the interesting property of the original anomaly mediation scenario. That is the approximated UV insensitivity due to the IR fixed point. This implies that the predicted anomaly induced masses in the  $N = 2$  sector, as well as the  $N = 2$  Yukawa couplings, do not change much even when there is some explicit breaking of  $\text{SO}(2)_R$  at the tree-level in the  $N = 2$  sector.

### 3 Application to Realistic Cases

#### 3.1 Translation from the Toy model to Realistic Models

In the following discussion, we suppose for simplicity that there is no particle in the  $N = 2$  sector both charged under  $\text{SU}(2)_L$  and  $\text{SU}(3)_c$ , e.g. left-handed squarks,  $Q_i$ , is assumed to be within the  $N = 1$  sector. Then although we have three kinds of gauge groups in the SM, the right-handed sleptons, left-handed sleptons and right-handed squarks in the  $N = 2$  sector are charged under  $\text{U}(1)_Y$ ,  $\text{SU}(2)_L \times \text{U}(1)_Y$  and  $\text{SU}(3)_c \times \text{U}(1)_Y$ , respectively. This justifies the application of the arguments in the previous section with two kinds of gauge groups as the effect of the third one is at a higher loop order.

We can consider the following correspondence between the toy model, Eq.(1), and a partially  $N = 2$  SSM.  $\text{SU}(N_c)$  corresponds to the  $\text{SU}(2)_L$  or  $\text{SU}(3)_c$  gauge group, while the  $\text{U}(1)$  corresponds to  $\text{U}(1)_Y$  of the SM gauge group.

We can divide the MSSM chiral multiplets into two sectors by some criterions, the  $N = 2$  sector and the  $N = 1$  sector, e.g. the left-handed smuon multiplet,  $L_2$ , the right-handed smuon multiplet,  $e_2$ , and the right-handed selectron multiplet,  $e_1$ , are within the  $N = 2$  sector while the others are within the  $N = 1$  sector. Then  $L_i$  in the toy model is assigned to be the chiral multiplets in the  $N = 2$  sectors of the MSSM, e.g.  $L_1$ ,  $L_2$ , and  $L_3$  in the toy model are assigned to be  $e_1$ ,  $e_2$  and  $L_2$ , respectively, in which case,  $N_F^S = 2$ . On the other hand,  $H_i$  in the toy model is assigned to be the multiplets in the  $N = 1$  sector, e.g.  $H_i$  are assigned to be the stops  $Q_3$ ,  $u_3$ , etc.

The chiral multiplets and their coupled gauge multiplets in the  $N = 2$  sector of the MSSM should have  $N = 2$  partners to construct the  $\text{SO}(2)_R$  symmetric gauge

interactions at the tree-level. Hence, we should introduce the hyperpartners,  $\overline{L}_i$ <sup>4</sup> (e.g.  $\overline{L}_2$ ,  $\overline{e}_1$  and  $\overline{e}_2$  are the hyperpartners of  $L_2$ ,  $e_1$  and  $e_2$ , respectively), and (some of) the vector partners (e.g.  $\phi_Y$ ,  $W$  are the  $N = 2$  vector partners of  $U(1)_Y$  and  $SU(2)_L$  gauge multiplets, respectively.) in addition to the MSSM particle contents. In general, the introduction of  $\overline{L}_i$  may lead to chiral gauge anomaly, and hence we should also introduce the spectator chiral multiplets to cancel the gauge anomaly, e.g.  $S_{e1}$ ,  $S_{e2}$ , and  $S_{L2}$  which have the same SM gauge representation with  $e_1$ ,  $e_2$ , and  $L_2$ , respectively. Then we obtain a well-defined field theory.

In this paper, we will consider the partially  $N = 2$  SSMs with restrictions of the gauge coupling unification and the perturbativity of dimensionless couplings. The precise gauge coupling unification is one of the prediction of the MSSM which implies that a gauge unified theory may exist beyond  $M_{GUT}$  leading to an explanation of the charge quantization. The requirement of the gauge coupling unification restricts the additional particle contents. We will take this restriction, but we do not suppose that the additional particles should be embedded in complete GUT multiplets, or the  $N = 2$  sector multiplets should be embedded into complete GUT multiplets. This is because the GUT breaking might be due to an orbifold projection in extra dimension and lead to missing GUT partners, which is a solution of the splitting doublet-triplet problem [18]. In fact, an extra dimension scenario is one of the leading candidates of the underlying theory for partially  $N = 2$  SSMs [9, 16].

### 3.2 Fixed Point Behavior in Partially $N = 2$ SSMs

In solving the tachyonic slepton problem in Sec.2.5, we have used an assumption that the  $N = 2$  Yukawa couplings can be approximated by their IR fixed point values Eqs.(31), (32), and (38). Here, we would like to further investigate on whether this assumption is realistic at  $m_{SUSY}$ .

There are several restrictions for the constants in a partially  $N = 2$  SSM.

**Chirality**  $F_1 \geq 6$  either for  $SU(2)_L$  or  $SU(3)_c$ .

The additional fermions should have mass terms to explain their absences in the experiments and hence have vector-like representations. Since the  $N = 2$

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<sup>4</sup>Instead of adding particles, we can also consider an  $N = 2$  hypermultiplet  $\{L_i, H_u\}$ , where  $L_i$  ( $H_u$ ) is a left-handed lepton (up-type Higgs) multiplet. In this case,  $\phi_Y$  is nothing but a right-handed neutrino multiplet. The large coupling in  $W = \sqrt{2}\tilde{Y}_i g_Y \phi_Y L_i H_u$  and light  $\phi_Y$  (or analogously  $W$ ) in this scenario give a too large neutrino mass  $> \mathcal{O}(\text{GeV})$  and hence is excluded [17].

sector fields can not be chiral while the SM fermions are chiral, the  $N = 1$  sector must contain the multiplets with the same representations as those of the SM fermions.

**Perturbativity**  $F_2 + F_1$  should be  $\lesssim 11(9)$  for  $SU(2)_L$  ( $SU(3)_c$ ).

The requirement of perturbative gauge couplings below  $M_{GUT}$  restricts the maximal number of the charged multiplets in a model.

From the first restriction, if a quark is in the  $N = 2$  sector, i.e.  $F_2$  of  $SU(3)_c$  is 1, we can find at the 1-loop level

$$\frac{d}{dt}g_3 \equiv \beta_3 \propto (F_1 + F_2 - 2 \times 3) > 0, \quad (47)$$

and hence

$$\alpha_{GUT} > \frac{g_c^2}{4\pi}. \quad (48)$$

is satisfied for both  $SU(2)_L$  and  $SU(3)_c$ . In other word, we always have the IR fixed point discussed in Sec.2.4 in realistic cases.

From Eqs.(36), (37), and (42), we can find that the exponents of them are essential for the convergence of the  $N = 2$  Yukawa couplings toward the fixed point values. Now we would like to estimate these exponents.

Neglecting the first term, which is a positive-semidefinite matrix if only fundamental representations,  $A_{i,j}$  in Eq.(35) leads to an exponent of Eq.(36),

$$\frac{\frac{F_1}{1 + \sum_{i=N_F^s+1}^{N_F} d(r_i)/(2(N_c^2-1))} + 4C(r_i)}{F_1 + F_2 - 2N_c} \quad (49)$$

$$> 13/14 \text{ (SU(2)}_L \text{ )} \quad (50)$$

$$> 320/33 \text{ (SU(3)}_c \text{ )}. \quad (51)$$

Here we have substituted the two restrictions and the derived relation  $\sum_{i=N_F^s+1}^{N_F} d(r_i) \leq 10$  (9) for  $SU(2)_L$  ( $SU(3)_c$ ) from perturbativity. In fact, by taking into account of  $\alpha_{GUT}$  with minimal possible additional particles, we can find for both  $SU(2)_L$  and  $SU(3)_c$ ,

$$\frac{\delta\omega_i^2(m_{SUSY})}{\delta\omega_i^2(M_{GUT})} \lesssim 10^{-1} \quad (52)$$

is satisfied. In the cases with only fundamental representations, the contribution of the first term in Eq.(35), which we have neglected, will increase the convergence when  $\delta\omega_i = \delta\omega_j$  for any  $i, j$ .

For the other  $N = 2$  Yukawa couplings, by a similar procedure, we obtain

$$\frac{\delta\tilde{Y}_i^2 g^2|_{m_{SUSY}}}{\delta\tilde{Y}_i^2 g^2|_{M_{GUT}}} \lesssim 10^{-1} \text{ for } i > N_F^s, \quad (53)$$

$$\frac{\delta\tilde{Y}_i^2(m_{SUSY})}{\delta\tilde{Y}_i^2(M_{GUT})} \lesssim 0.7 \text{ for } i \leq N_F^s. \quad (54)$$

In Eq.(54), we have assumed that the singlets have hypercharges,  $Y_i = 1$  and consider a specific case shown in the footnote 5 which should have the weakest convergence due to the small gauge couplings.

Since the difference between a tree-level  $N = 2$  Yukawa coupling and its fixed point value is

$$\delta\omega_i^2(M_{GUT}) \sim \delta\tilde{Y}_i^2(M_{GUT}) \sim \mathcal{O}(1). \quad (55)$$

In partially  $N = 2$  SSMs, the assumption made in Sec.2.5 is satisfied at less than 10% level in general for the  $N = 2$  Yukawa couplings of non-singlets under  $SU(2)_L$  or  $SU(3)_c$ . However, for singlets this assumption is a rough approximation.

Therefore, the argument in Sec.2.5 is satisfied for non-singlet  $N = 2$  hypermultiplets of  $SU(2)_L$  or  $SU(3)_c$  in the  $N = 2$  sector. This implies that the anomaly induced mass squares are positive and approximately UV insensitive. For right-handed sleptons in the  $N = 2$  sector, the anomaly induced mass is a rough approximation but we find that the tachyonic slepton problem is also solved in most of the cases.<sup>5</sup>

### 3.2.1 An Example Model and the Phenomenological Aspects

Now we would like to consider a simplest model with a non-abelian  $N = 2$  vector partner at the SUSY scale. In fact, due to the small number of the particle contents, this model has the weakest convergence toward the fixed point among the models with non-singlet hyperpartners of  $SU(2)_L$  or  $SU(3)_c$  in the  $N = 2$  sector.

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<sup>5</sup> In particular, we have checked the most UV sensitive case where there are two  $(\{\bar{\mathbf{3}}, 1, 1/3\} + h.c.)$ , one  $(\{1, 1, -1\} + h.c.)$ , and one  $\{1, \mathbf{3}, 0\}$  in addition to the MSSM particles. Here,  $\{\}$  denotes the representation of  $\{SU(3)_c, SU(2)_L, U(1)_Y\}$ , respectively.  $\{1, 1, -1\}$  is assigned to be the hyperpartner of the right-handed smuon. The gauge couplings unify with  $\alpha_{GUT} \sim 1/18$  and the negative anomaly induced mass squared of this smuon becomes positive below the scale around  $10^7 \text{ GeV}$ . The tachyonic slepton problem is solved. Since the other cases have better convergence to the fixed point value due to Eq.(42) and the condition of perturbative gauge coupling unification, the tachyonic slepton problem of a right-handed slepton should be solved in general. We have not encounter any exception by using RG equations and anomalous dimensions at the 2-loop level.



We have additional hyperpartners  $\bar{L}_2, \bar{e}_2$  and  $\bar{e}_1$  for  $L_2, e_2$ , and  $e_1$ , respectively. To have an  $\text{SO}(2)_R$  symmetry at the tree-level, we also introduce two vector-partners  $W$  and  $\phi_Y$  corresponding to  $\text{SU}(2)_L$  and  $\text{U}(1)_Y$  gauge groups, respectively. The spectators,  $S_{L1}, S_{e1}$  and  $S_{e2}$ , are introduced to cancel the gauge anomaly induced by the additional hyperpartners. Also introduced is an octet chiral multiplet  $G$  to satisfy the gauge coupling unification at  $M_{GUT}$  with  $1/\alpha_{GUT} \sim 11.5$ . The superpotential is given as follows.

$$W = W_{N2} + W_{\text{mass}} + W_{\text{MSSM}}, \quad (56)$$

$$W_{N2} = \sqrt{2}\bar{L}_2(\omega_L g_2 W + \tilde{Y}_L g_Y \phi_Y)L_2 + \sqrt{2}\sum_i^2 \bar{e}_i g_Y \tilde{Y}_{e_i} \phi_Y e_i \quad (57)$$

$$W_{\text{mass}} = -M_G \text{tr}[G^2] - M_W \text{tr}[W^2] - M_Y \phi_Y^2 - M_L \bar{L}_2 S_{L2} - \sum_i^2 M_{e_i} \bar{e}_i S_{e_i} \quad (58)$$

$$W_{\text{MSSM}} \sim y_t H_u Q_3 u_3 + y_b H_d Q_3 d_3 + y_\tau H_d L_3 e_3 + \mu H_u H_d. \quad (59)$$

Here  $y_t, y_b, y_\tau$ , and  $\omega_L, \tilde{Y}_{e1, e2, L}$  are the MSSM top-Yukawa, bottom-Yukawa, tau-Yukawa couplings, and the  $N = 2$  Yukawa couplings, respectively.  $d_3, L_3$ , and  $e_3$  are the right-handed bottom squark, left-handed tau lepton, and right-handed tau lepton multiplets, respectively.  $\mu$  is the Higgs mixing  $\mu$ -term.

We have shown in Sec.2.3, the tree-level conditions of the  $\text{SO}(2)_R$  and the vanishing gaugino masses in the  $N = 2$  sector lead to a sequestering of this sector from the SUSY breaking:

$$\omega_L = 1, \tilde{Y}_L = 1/2, \tilde{Y}_{e1} = \tilde{Y}_{e2} = 1, m_{\tilde{e}_1}^2 = m_{\tilde{e}_2}^2 = m_{L_2}^2 = 0, \quad (60)$$

$$M_1 = M_2 = 0, m_W^2 = m_{\phi_Y}^2 = 0, \quad (61)$$

and the vanishing  $A$ -terms in the  $N = 2$  sector. We further suppose that the  $D - (S-)$  term vanishes.

We calculate the 2-loop RG equations by using the boundary condition of Eq.(60) and obtain the RG running illustrated in the left-hand side of Fig.1. In this figure, the gray solid lines represent the scale dependence of the SM gauge couplings  $\{\sqrt{\frac{5}{3}}g_Y, g_2, g_3\}$ , and the  $\{\text{red dotted (black dotted), green dashed (zero-axis line), blue dot-dashed (black dot-dashed)}\}$  lines represent that of  $\{g_2\omega_L, 2\sqrt{\frac{5}{3}}g_Y\tilde{Y}_L, g_Y\sqrt{\frac{5}{3}}\tilde{Y}_{e1, e2}\}$  (at the fixed point), respectively. We can find that the  $N = 2$  Yukawa couplings approach to their fixed point values. The convergence of the fixed point is shown in the left-hand side of Fig. 2 which also implies the UV insensitivity of the  $N = 2$  Yukawa

couplings at the low energy. This figure represents the RG running of the  $N = 2$  Yukawa couplings with varying the tree-level value of the running Yukawa coupling  $\pm 20\%$  from that of Eq.(60), while keeping other couplings unchanged. The {red solid, green dotted, blue dashed} lines represent the running of  $\{\omega_L, \tilde{Y}_L g_Y, -\tilde{Y}_{e1,e2}\}$ , respectively. The black solid lines denote the fixed point values evaluated by Eqs.(31), (32), and (38).

In particular, the numerically evaluated values of the couplings at the renormalization scale,  $\mu_{RG} = 10\text{TeV}$ , are

$$\omega_L^2 \sim 2.413, \quad g_Y^2 \tilde{Y}_L^2 \sim 0.045, \quad \tilde{Y}_{e1}^2 \sim \tilde{Y}_{e2}^2 \sim 1.93, \quad (62)$$

with

$$g_Y^2 \sim 0.136, \quad g_2^2 \sim 0.407, \quad g_3^2 \sim 0.929. \quad (63)$$

Here, we haven't matched the gauge couplings with the ones in the SM precisely as it depends on the spectrum in the  $N = 1$  sector and also the SUSY mass in the  $N = 2$  sector. These mass terms depend on the UV physics which we do not specify. Since the  $\mathcal{O}(1)$  changes of  $y_t, y_b$  and  $y_\tau$  correct Eq.(62) up to 1%, we also do not specify them.

In the right-hand side of Fig. 1, the scale dependence of some relevant anomaly induced masses is shown with  $m_{3/2} = 100\text{TeV}$  at the 2-loop level. The gray solid and dotted lines represent the scale dependence of bino and wino masses, while the {red solid (black solid), green dashed (black dashed)} lines represent the masses of the  $\{m_{\tilde{L}2}, m_{\tilde{e}1,\tilde{e}2}\}$  (at the fixed point), respectively. Here, the sign of the mass denotes the corresponding sign of the mass squared. We can find below  $\mu_{RG} = 10^9\text{GeV}$  the tachyonic slepton problem is solved and the anomaly induced masses of the sleptons approach to the fixed point values. In the right-hand side of Fig. 2, we express the UV insensitivity of these anomaly induced masses. The running of these masses with different boundary conditions of the  $N = 2$  Yukawa couplings are shown. The {red solid, green dashed} lines represent the RG running of  $\{m_{\tilde{L}2}^2, m_{\tilde{e}1,\tilde{e}2}^2\}$ , respectively. The {black solid, black dashed} lines denote the fixed point values evaluated using Eqs.(31), (32), and (38).

Therefore, in the  $N = 2$  sector, the  $N = 2$  Yukawa interactions are approximately expressed by the IR gauge couplings, and hence the anomaly induced soft masses are approximately represented by only one free parameter,  $m_{3/2}$ .

In particular, the numerical values of the relevant anomaly induced masses in the

$N = 2$  sector are evaluated as follows, where  $\mu_{RG} = 10$  TeV,

$$m_{\tilde{e}1} = m_{\tilde{e}2} \sim 0.00354m_{3/2}, \quad m_{\tilde{L}2} \sim 0.00556m_{3/2}, \quad (64)$$

$$M_1 \sim 0.0139m_{3/2}, \quad M_2 \sim 0.0108m_{3/2}. \quad (65)$$

Such sleptons might be excluded up to 250GeV (350GeV) for right-handed ones (left-handed one)[27]<sup>6</sup>. Therefore, we may have a constraint,

$$m_{3/2} > 70\text{TeV}. \quad (66)$$

On the other hand, the mass scales of the  $N = 1$  sector, which is not restricted by  $\text{SO}(2)_R$  symmetry, can be  $\mathcal{O}(10)\text{TeV}$  depending on the UV physics. The RG contribution to the  $N = 2$  sector scalar soft masses proportional to a mass of this order in the  $N = 1$  sector can be subdominant to Eq.(64) with  $m_{3/2} = \mathcal{O}(100)$  TeV, as it is a 2-loop-level effect and proportional to either  $g_2^2$  or  $g_Y^2$ .

Therefore, we should be allowed to have

$$m_{\tilde{L}, \tilde{L}_R} \gtrsim 6\text{TeV}, \quad (67)$$

without spoiling our predictions. This is actually the range where the Higgs boson mass is explained [26]. Moreover, if the  $\mu$ -term, and the ratio of the vacuum expectation value of  $H_u$  to  $H_d$ , i.e.  $\tan \beta$ , are large enough, the muon  $g - 2$  contribution can be approximately evaluated as [7, 24],

$$\delta\alpha_\mu \simeq \left( \frac{1}{1 + \Delta_\mu} \right) \frac{g_Y^2}{16\pi^2} \frac{m_\mu^2 \mu \tan \beta M_1}{m_{\tilde{L}2}^2 m_{\tilde{e}2}^2} f_N \left( \frac{m_{\tilde{L}2}^2}{M_1^2}, \frac{m_{\tilde{e}2}^2}{M_1^2} \right), \quad (68)$$

$$= 24.9 \times 10^{-10} \left( \frac{1.391}{1 + \Delta_\mu} \right) \left( \frac{\mu \tan \beta}{300\text{TeV}} \right) \left( \frac{80\text{TeV}}{m_{3/2}} \right)^2, \quad (69)$$

where,

$$\Delta_\mu \simeq \mu \tan \beta \frac{g_Y^2 M_1}{16\pi^2} I(M_1^2, m_{\tilde{L}2}^2, m_{\tilde{e}2}^2) = 0.391 \left( \frac{\mu \tan \beta}{300\text{TeV}} \right) \left( \frac{80\text{TeV}}{m_{3/2}} \right). \quad (70)$$

Here,  $I(x, y, z)$  and  $f_N(x, y)$  are loop functions which can be found in the references. If we quote [4, 5]

$$\delta\alpha_{\text{exp}} = (26.1 \pm 8.0) \times 10^{-10}. \quad (71)$$

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<sup>6</sup>The process we consider can be either slepton decaying to lepton and neutrino (R-parity violation) or slepton decaying to lepton and lightest singlet (might-be bini or wini). We have assumed the decoupling of the other particles than that in the process. For left-handed smuon, the bound should be over-estimated due as we have only one light flavor. To the case of long-lived slepton the bounds become a bit severer [28].

as a reference value of the experimental deviation from the SM prediction, The muon  $g - 2$  anomaly can be explained within the  $1\sigma$  level error ( $2\sigma$  level error) when  $m_{3/2} \leq 100$  TeV (120 TeV) with  $\mu \tan \beta < 500$  TeV.

In this model if  $\mu \sim m_{SUSY} \sim \mathcal{O}(10)$  TeV, the muon  $g - 2$  anomaly can be explained with a cosmologically favored gravitino mass range,  $m_{3/2} \sim 100$  TeV. In fact, the cosmological gravitino problem is alleviated due to its short life time [19]. Also, in our scenario, the SUSY breaking field is not necessarily a gauge singlet, and in the case we do not have cosmological Polonyi problem [20].

On the other hand, with a large  $\mu$ -term, we can not suppose that a neutralino in the MSSM is the DM as the neutralinos are all heavier than the smuons. Instead, naturally we can introduce several new DM candidates in the additional particle contents. This is because we have a theoretical reason to have a stable additional particle. Since we have a singlet chiral multiplet,  $\phi_Y$ , there is a serious tadpole problem [25], e.g. supergravity can generate a dangerous tadpole term as  $W \sim m_{3/2} M_{GUT} \phi_Y$ . A simple way to solve this problem is to impose a precise  $Z_2$  symmetry, under which  $\phi_Y$  is odd, and this term is forbidden. Then, we can find from Eq.(57),  $W$  should also be  $Z_2$  odd. Since this symmetry should be a precise one, the lightest  $Z_2$  odd particle should be stable. Hence, if a component of,  $\phi_Y$  and  $W$ , is the lightest  $Z_2$  odd particle, it might be the DM. The lightest  $Z_2$  odd particle depends on the SUSY mass Eq.(58) and the corresponding “ $b$ -term”. Since this  $Z_2$  is not necessarily the  $R$ -parity, the DM is not constrained to be lighter than the smuons if the  $R$ -parity is violated. In this case, the cosmological gravitino problem might be further alleviated. This is because the decay products of a gravitino do not always eventually decay into a DM, and the overproduction of the DM due to the gravitino decay should be alleviated.

In general, since some of the MSSM chiral multiplets are within the  $N = 2$  sector, we must introduce a singlet to compose an  $N = 2$  gauge interaction of  $U(1)_Y$ . Hence, from the same argument, we should have a stable  $Z_2$  odd particle also in other partially  $N = 2$  SSMS.

The study of cosmology for partially  $N = 2$  SSMS will be given elsewhere.

## 4 Conclusions

To explain the tension between the measured Higgs boson mass and some of the deviations in flavor physics, we consider partially  $N = 2$  SUSY extensions of the

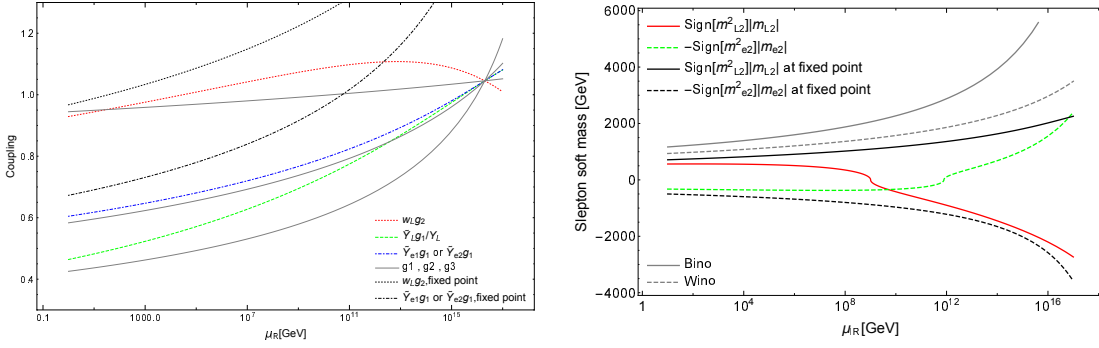


Fig. 1: (Left) The 2-loop RG running of the gauge and Yukawa couplings. (Right) The 2-loop RG running of the anomaly induced masses of the MSSM particles in the  $N = 2$  sector. The sign of the mass in the figure represents the sign of the mass square. The detailed explanations are in the text.

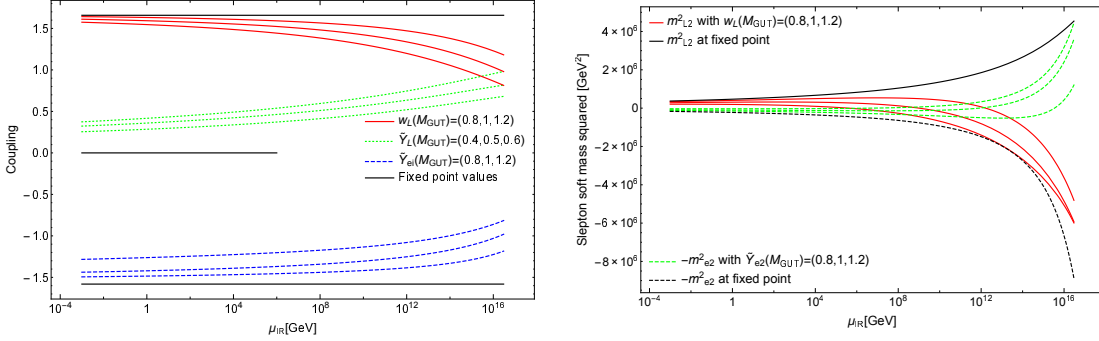


Fig. 2: (Left) The UV insensitivity of the  $N = 2$  Yukawa couplings. (Right) The UV insensitivity of the anomaly induced mass squares in the  $N = 2$  sector. In both figures, for illustrative purpose, we flip the sign for some parameters. The detailed explanations are in the text.

MSSM (partially  $N = 2$  SSMS). The tree-level  $\text{SO}(2)_R$  symmetry imposed in the  $N = 2$  SUSY sector ( $N = 2$  sector), in turns sequesters this sector from the SUSY breaking at the tree-level. We have studied the IR behavior within this sequestered sector, which is almost independent of the UV physics and the spectra in the  $N = 1$  SUSY sector ( $N = 1$  sector). We consider the stops are within the  $N = 1$  sector and hence can be heavy enough to explain the measured Higgs boson mass.

In the  $N = 2$  sector, the radiative corrections alter the tree-level  $\text{SO}(2)_R$  symmetry drastically in two perspectives: the RG running of the Yukawa couplings towards a fixed point, and the anomaly induced soft masses of the scalars and gauginos. The fixed point values of the Yukawa couplings are non-trivially large due the lack of the

$N = 2$  sector fields contrary to those in the  $N = 1$  sector. We have found in general, at low energy the anomaly induced scalar mass squares turn out to be positive due to the large Yukawa couplings, and solving the tachyonic slepton problem. Due to the strong convergence of the fixed point, the low energy anomaly induced masses as well as the Yukawa couplings are approximated by the fixed point values. This implies, in the  $N = 2$  sector the Yukawa couplings are almost determined and the soft masses are almost expressed by one parameter, the gravitino mass. Also, our result should hold even with a small explicit breaking of the  $SO(2)_R$  symmetry at the tree-level.

We propose a concrete model as an example of partially  $N = 2$  SSMs. Other than confirming the above facts, we have shown that the muon  $g - 2$  anomaly can be explained within its  $1\sigma$  level error with the gravitino mass,  $\mathcal{O}(100)\text{TeV}$ , and the SUSY scale,  $\mathcal{O}(10)\text{TeV}$ . We have argued the cosmological aspects, where we have proposed new DM candidates as a natural prediction of partially  $N = 2$  SSMs. Also, some phenomenological aspects are discussed.

## 5 Discussions

Let us comment on some problems and their possible solutions of the example model in Sec.3.2.1 (Some of them are generic in the partially  $N = 2$  SSMs).

One is the lepton flavor violation problem, namely the question why the introduced hyperpartners preserve the muon and electron numbers. This problem might be solved by considering some suitable UV physics. Alternatively, we can introduce more hyperpartners of the sleptons such that the  $N = 2$  sector is automatically flavor symmetric.

The second is the possibility of the too small gluino mass. However, if at the tree-level majorana gaugino masses are forbidden by the fact that there is no singlet SUSY breaking field,  $Z$ , the gluino majorana mass should be also vanish at the tree-level and is dominantly anomaly induced. Since  $\frac{d}{dt}g_3 \sim 0$  at the 1-loop level, the 2-loop anomaly induced mass at  $\mu_{IR} = 10\text{TeV}$

$$M_3(10 \text{ TeV}) \sim 0.00230m_{3/2}, \quad (72)$$

might be too light to survive the experimental constraints, where a R-parity violating gluino is excluded up to 1080 GeV and a long-lived one is excluded up to  $\sim 1500$  GeV

[29]. This at least requires  $m_{3/2} \gtrsim 500$  TeV, in which case the muon  $g - 2$  anomaly is no longer explained. There are two ways to tackle this situation. One is to have a Dirac gluino mass term, as  $W = \frac{D^a}{M_{GUT}} \text{tr}[GW_a^{(\text{SU}(3))}]$ , where  $D^a = D_Z \theta^a$  and  $W_a^{(\text{SU}(3))}$  are an additional spurion SUSY breaking field<sup>7</sup> and the field strength of the  $\text{SU}(3)_c$  gauge interaction, respectively. Notice that such a Dirac mass term is not allowed for bino or wino due to the  $Z_2$  parity, and our prediction would not change unless  $D_Z$  is extremely large. The other way is to have a large  $M_G$  with a supergravity induced “ $b$ -term”,  $V = M_G m_{3/2} \text{tr}[GG]$ . Then the decoupling of  $G$  induces a gauge mediation effect to raise the gluino mass to be the MSSM anomaly induced one,  $\gtrsim 2\text{TeV}$ , with  $m_{3/2} \gtrsim 100\text{TeV}$  without changing any other results at the leading order.

Also, we have some remaining theoretical questions. We haven’t asked the origin of the  $\text{SO}(2)_R$  violating SUSY mixing masses,  $M_G$ ,  $M$ , and  $M_{Li}$ , in Eq.(1) and their corresponding SUSY breaking mixing masses. Since a partially  $N = 2$  SSM is an effective theory with explicit breakings of the  $\text{SO}(2)_R$ , we should also ask whether there is such a natural UV theory with an exact  $\text{SO}(2)_R$  symmetry (or  $N = 2$  SUSY), that generates the partially SSM with well-approximated Eqs.(20), (21), and (22)<sup>8</sup>.

However, even if there is not such an UV theory and Eqs.(20), (21), and (22) are violated badly, the fixed point behavior proposed here still has predictive powers for this MSSM with additional matters due to the strong convergence.

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<sup>7</sup>This is a natural introduction when the SUSY breaking  $N = 2$  multiplet is a vector multiplet, as  $D^a$  can be considered as field strength for the vector partner of the chiral SUSY breaking field,  $Z$ .

<sup>8</sup>We will show that the set-up can be generated from a simple extra-dimensional theory in [16].

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